

Exercise 53

Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

$$x^4 + x - 3 = 0, \quad (1, 2)$$

Solution

Let $f(x) = x^4 + x - 3$. This is a polynomial function, which is continuous according to Theorem 7.

$$f(x) = 0, \quad (1, 2)$$

Find a value of x in the interval $[1, 2]$ so that $f(x)$ is negative, and find a value of x in the interval $[1, 2]$ so that $f(x)$ is positive.

$$f(1) = -1$$

$$f(2) = 15$$

$f(x)$ is continuous on the closed interval $[1, 2]$, and $N = 0$ lies between $f(1)$ and $f(2)$. By the Intermediate Value Theorem, then, there exists a number c such that $f(c) = 0$.